Transition to a Two-Level Linear State Estimator, part II: Algorithm

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Abstract-- The availability of synchro-phasor data has raised the possibility of a linear state estimator if the inputs are only complex currents and voltages and if there are enough such measurements to meet observability and redundancy requirements. Moreover, the new digital substations can perform some of the computation at the substation itself resulting in a more accurate two-level state estimator. The main contribution in this paper is that this two-level processing removes the bad data and topology errors, which are major problems today, at the substation level.

In Part I of the paper, we describe the layered architecture of databases, communications, and the application programs that are required to support this two-level linear state estimator. In Part II, we describe the mathematical algorithms that are different from those in the existing literature.

As the availability of phasor measurements at substations will increase gradually, this paper describes how the state estimator can be enhanced to handle both the traditional state estimator and the proposed linear state estimator simultaneously. This provides a way to immediately utilize the benefits in those parts of the system where such phasor measurements become available and provides a pathway to transition to the 'smart' grid of the future.

Index Terms—**PMU, State Estimation, Energy control centers.**

I. Introduction

The traditional state estimator function is executed centrally in a control center by gathering all the data needed, real time and static, to solve the Topology Processor (TP), State Estimation (SE) and the Bad Data Detection-Identification (BD) sequentially. In Part I, we have presented a two-level communications architecture in which the data and the calculations are distributed among the substations and the control center. The main assumption in this paper is that enough phasor measurements are available at each substation to provide observability and redundancy for a linear state estimator introduced in [1], [2].

In this paper, we propose a distributed two level state estimator function that uses a linear SE algorithm at the control center level but moves the local topology processing and the bad data detection-identification to the substation level. The local topology processing and bad data detection-identification at each substation is done with very different algorithms than is used today at the control center level. In fact, substation level topology processing and bad data detection-identification are no longer separate functions but analog measurement errors and circuit breaker status errors are identified together. The result is that both the topology results and the measurement data are filtered locally to provide a much more accurate set of data to the control center level state estimation calculation. Unlike the present state estimator and some other distributed state estimators [3]-[11], the bad data detection-identification and filtering is done prior to the state estimation solution rather than after. The linearity of the state estimator guarantees a solution and the substation level pre-filtering of topology and measurement errors guarantee the accuracy of the solution.

In this Part II of the paper, the algorithms are described first followed by some experimental results. In the next section, the linear state estimation formulation is described briefly for the sake of completeness. In section III, we describe the new substation level calculations that pre-filter all the real-time data at that substation; we call this the Substation Level State Estimator or Zero Impedance State Estimator. In section IV, we briefly describe the rest of the linear algorithm that is executed in the control center. In section V, we formulate the transitional hybrid algorithm when this new algorithm must co-exist with the traditional algorithm as existing substations are transitioned into the new architecture. Finally, we show some experimental results for the substation level calculations in section VI.

II. Linear state estimator

As introduced in [1], [2], if all the analog measurements were synchronized currents and voltages, then the state estimation equations would be linear. With the help of increasing installations of phasor measurements, we can implement a new measurement function of the state estimator which is linear in the complex plane. In this state estimator, both the states and the measurements are defined in the complex plane and the measurement functions are linear, making this a linear state estimator.

So the linear state estimation in the complex plane is the following optimization problem:

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$$\text{Min } \tilde{\mathbf{r}}^T \tilde{\mathbf{W}} \tilde{\mathbf{r}} \quad \text{s.t. } \tilde{\mathbf{z}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{r}} \tag{1}$$

where $\tilde{\mathbf{r}}$ is the residuals vector, $\tilde{\mathbf{W}}$ is the weight matrix, $\tilde{\mathbf{z}}$ is the measurements vector, $\tilde{\mathbf{x}}$ is the vector of system states, and $\tilde{\mathbf{H}}$ is the measurement function matrix relating the measurement vector to the states.

For computational purposes, each measurement $\tilde{z}_i = z_{i,real} + jz_{i,imag}$, each state $\tilde{x}_i = x_{i,real} + jx_{i,imag}$, and each residual $\tilde{r}_i = r_{i,real} + jr_{i,imag}$ can be represented as the 2×1 vector $\mathbf{z}_i = \begin{bmatrix} z_{i,real} \\ z_{i,imag} \end{bmatrix}$, $\mathbf{x}_i = \begin{bmatrix} x_{i,real} \\ x_{i,imag} \end{bmatrix}$, and $\mathbf{r}_i = \begin{bmatrix} r_{i,real} \\ r_{i,imag} \end{bmatrix}$ in the real plane respectively. For each entry $\tilde{h}_{i,j} = h_{i,j,real} + jh_{i,j,imag}$ in the measurement function matrix $\tilde{\mathbf{H}}$, a 2×2 matrix $\mathbf{H}_{i,j} = \begin{pmatrix} h_{i,j,real} & -h_{i,j,imag} \\ h_{i,j,imag} & h_{i,j,real} \end{pmatrix}$ can represent it in the real plane. Then, if there are m measurements and n states in the system, (1) can be written as [2]:

$$\text{Min } \mathbf{r}^T \mathbf{W} \mathbf{r}$$
$$\text{s.t. } \mathbf{z} = \begin{pmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_m \end{pmatrix} = \mathbf{Hx} + \mathbf{r} = \begin{pmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,n} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{m,1} & \cdots & \mathbf{H}_{m,n} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{pmatrix} + \begin{pmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_m \end{pmatrix} \tag{2}$$

where $\mathbf{W}$ is the diagonal weight matrix in which all the entries are real numbers and $\mathbf{W}_i$ is the 2×2 weight block for each measurement:

$$\mathbf{W}_i = \begin{pmatrix} \sigma^2_{z_{1,real}} & 0 \\ 0 & \sigma^2_{z_{1,real}} \end{pmatrix}$$

For the above linear state estimation problem, the solution is obtained [2] without iteration:

$$\mathbf{x} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{W}\mathbf{z} \tag{3}$$

In the following two sections, we develop the state estimator formulations at both the substation and control center levels. All the formulations are linear and use (3) to obtain the solution.

As in the traditional state estimator, this linear state estimator requires sufficient redundancy for the solution to provide good estimates. Also, observability is required to obtain a solution and thus on-line observability checking before every execution of the state estimator has to be part of the calculation cycle. Because the redundancy and observability calculations for this linear SE are similar to that of the traditional SE [1], we have not addressed these calculations further in this paper.

III. Substation Level Linear State Estimator

In [12]-[15], many substation level data processing methods are provided. Here we use a novel but simple method to do the substation level state estimation. The substation level state estimator is solved in several parts. First, each voltage level in the substation is solved separately. The advantage is that the substation circuit at one voltage level has no impedances, thus simplifying the SE equations. Second, the current phasor measurements and the voltage phasor measurements are handled separately. This further simplifies the equations, but more than that the SE based on current phasor measurements is used to determine the circuit breaker statuses. Of course, the results are finally combined at the control center level.

a) Zero Impedance Current State Estimator

In this part, we use all the current phasor measurements to estimate the circuit breaker currents, which are the states of this SE formulation. These circuit breaker currents can then also be used to determine the circuit breaker statuses. In this model, all the bus sections and circuit breakers at the same voltage level inside the substation construct a zero impedance power system.

First, we assume the availability of current phasor measurements on each circuit breaker and branch to get the measurements sets $\mathbf{z}_{cb}$ and $\mathbf{z}_{inj}$ respectively. We also assume that all the circuit breakers are closed to start with (the solution will identify the correct circuit breaker statuses). The equations for the branch current measurements can then be written as follows:

$$\mathbf{z}_{inj} = \mathbf{A}_{KCL}\mathbf{x} + \mathbf{r}_{inj} \tag{4}$$

where $\mathbf{z}_{inj}$ is the injection current at each node, $\mathbf{A}_{KCL}$ is the incidence matrix connecting bus sections to circuit breakers in the zero impedance power system, $\mathbf{x}$ is the state vector of the circuit breaker currents, and $\mathbf{r}_{inj}$ is the corresponding residual vector.

The equations for the current measurements $\mathbf{z}_{cb}$ at each circuit breaker have the obvious relationship:

$$\mathbf{z}_{cb} = \mathbf{I}\mathbf{x} + \mathbf{r}_{cb} \tag{5}$$

where $\mathbf{I}$ is the identity matrix, and $\mathbf{r}_{cb}$ is the corresponding residual vector.

Then the measurements functions can be represented by:

$$\mathbf{z} = \begin{pmatrix} \mathbf{z}_{inj} \\ \mathbf{z}_{cb} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{KCL} \\ \mathbf{I} \end{pmatrix} \mathbf{x} + \begin{pmatrix} \mathbf{r}_{inj} \\ \mathbf{r}_{cb} \end{pmatrix} = \mathbf{Hx} + \mathbf{r} \tag{6}$$

This is a simple linear state estimation problem (the entries in H matrix are 1, 0, or -1), so we can find the estimation solution by (3). Once the circuit breakers currents are estimated, the analog bad data can be identified and rejected by the traditional testing method based on largest normalized residual. The zero impedance current state estimation is repeatedly executed until no bad data remains.

The final estimated circuit breaker currents can then be directly utilized to verify the digital status of the corresponding circuit breakers to identify any topology errors. For example, if the estimated current for a circuit breaker is not close to zero but the digital measurement of this circuit breaker is open, we conclude with high probability that the status measurement is bad and the real state of the circuit breaker is closed. A special case is when the estimated breaker current is close to zero and the status measurement indicates a closed breaker; in such a case one cannot conclude with high probability the status of the breaker. Fortunately, this status does not affect our estimate calculations but may impact other operator decisions.

After the bad status data are eliminated, we need to repeat this algorithm again because the new topology will give us a new incidence matrix $\mathbf{A}_{KCL}$ to enable a more precise estimation of the currents.

We can see from this algorithm that the analog and the digital states are decoupled, i.e. the analog estimation can be done without knowing the digital status correctly as long as there is enough redundancy which means such analysis is possible only if there are redundant current phasor-measurements in the substation. The advantage of this is we can identify the bad data for not only the analog data but also for the circuit breaker and switch statuses directly. In the traditional SE there is no effective method for identifying status errors, which have become a major source of errors in the state estimators today.

It should be pointed out that one of the major advantages of this two-level state estimator is that the error detection and identification is done at the substation level. This is many times more efficient than the present method because each calculation at a substation is small and moreover, is done in parallel. Of course, the ability to detect status errors is another major advantage.

b) Zero Impedance Voltage State Estimator

After we get the estimated digital status of each circuit breaker, we can construct the topology for this voltage level. The outputs will include the number of buses at this voltage level and all its connected elements and branches. We can then estimate the bus voltage from the voltage measurements at all the bus sections comprising this bus. This is essentially a weighted average and is formulated here as a zero impedance voltage state estimator. The states are the voltage of each bus, and the measurements are the voltage phasor measurements at the bus sections belonging to the bus. The measurement function then is:

$$\tilde{\mathbf{z}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{r}} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \tilde{\mathbf{x}} + \tilde{\mathbf{r}} \tag{7}$$

Because $\mathbf{H}$ is actually a vector with all 1s, the actual solution is the diagonal weighted average value of all the measurements:

$$x_{real} = \frac{\sum_{i=1}^{m} w_{2i-1,2i-1} z_{i,real}}{\sum_{i=1}^{m} w_{2i-1,2i-1}} \tag{8}$$

$$x_{imag} = \frac{\sum_{i=1}^{m} w_{2i,2i} z_{i,imag}}{\sum_{i=1}^{m} w_{2i,2i}} \tag{9}$$

where $w_{i,i}$ is the i th diagonal entry of $\mathbf{W}$.

Although the substation level state estimator proposed here has several steps, the calculations are very simple and fast. At the end, this calculation can output any analog values based on the estimated states at each substation together with the substation topology. At a minimum, we assume that branch currents, bus injection currents, and bus voltages for each voltage level will be transferred to the control center. The main advantages of this substation level state estimator are:

(1) It utilizes all of the current and voltage measurements in the substation and provides a more accurate phasor measurement set to the upper level applications.

(2) It provides a direct way to estimate the status of circuit breakers from analog measurements to avoid topology errors.

IV. Control center level linear state estimator

The control center level state estimator receives all the analog estimated measurements - bus voltages, branch currents, and injection currents - and the substation topology from the substation level state estimators. This SE then connects all the substation topologies with the branch data to get the whole system topology.

Based on the idea introduced in [2], given a power system with n buses and m branches, the measurement inputs are the bus voltages $\tilde{\mathbf{V}}_{bus} = [\tilde{v}_{bus,1}, \ldots, \tilde{v}_{bus,n}]^T$, branch currents at both ends $\tilde{\mathbf{I}}_{b1} = [\tilde{i}_{b,1}, \ldots, \tilde{i}_{b,m}]^T$, $\tilde{\mathbf{I}}_{b2} = [\tilde{i}_{b,m+1}, \ldots, \tilde{i}_{b,2m}]^T$, and the injection currents $\tilde{\mathbf{I}}_{inj} = [\tilde{i}_{inj,1}, \ldots, \tilde{i}_{inj,m}]^T$. The states of the system are the bus complex voltages $\tilde{\mathbf{x}} = [\tilde{v}_1, \ldots, \tilde{v}_n]^T$. Assuming the system admittance matrix is:

$$\tilde{\mathbf{Y}} = \begin{pmatrix} \tilde{y}_{11} & \cdots & \tilde{y}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{y}_{n1} & \cdots & \tilde{y}_{nn} \end{pmatrix} = \begin{pmatrix} g_{11} + jb_{11} & \cdots & g_{1n} + jb_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} + jb_{n1} & \cdots & g_{nn} + jb_{nn} \end{pmatrix}.$$

The measurement function can then be written as:

$$\tilde{\mathbf{z}} = \begin{pmatrix} \tilde{\mathbf{V}}_{bus} \\ \tilde{\mathbf{I}}_{b1} \\ \tilde{\mathbf{I}}_{b2} \\ \tilde{\mathbf{I}}_{inj} \end{pmatrix} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{r}} = \begin{pmatrix} \mathbf{I} \\ \tilde{\mathbf{Y}}_1^b \\ \tilde{\mathbf{Y}}_2^b \\ \tilde{\mathbf{Y}} \end{pmatrix} \tilde{\mathbf{x}} + \tilde{\mathbf{r}} \tag{10}$$

where $\tilde{\mathbf{Y}}_1^b$, $\tilde{\mathbf{Y}}_2^b$ can be derived from the branch admittances.

Using real number vectors and blocks to represent each complex number, the measurement function should have the form:

$$\mathbf{z}=\begin{pmatrix}\mathbf{V_{bus}}\\ \mathbf{I_{b1}}\\ \mathbf{I_{b2}}\\ \mathbf{I_{inj}}\end{pmatrix}=\mathbf{Hx}+\mathbf{r}=\begin{pmatrix}\mathbf{I}\\ \mathbf{Y_1^b}\\ \mathbf{Y_2^b}\\ \mathbf{Y}\end{pmatrix}\mathbf{x}+\mathbf{r} \tag{11}$$

The control center level state estimator formulation then is still another linear state estimator problem (11).

Compared with traditional state estimator, our control center level state estimator has much less of a computational burden. The system topology build by merging all the substation topology together is much faster as most of the topology processing is done at the substations. The SE solution is much faster as most of the estimation is now done at the substation level and the linearity guarantees a solution with no divergence. At the same time, both the system topology and the input measurements are more reliable because they have already been estimated at the substation level. Any bad data detection or identification done at the control center should be less burdensome.

This two-level linear SE has many advantages over the traditional SE but it assumes the availability of enough phasor measurements that provides observability and redundancy at all substations. Although this is not the case today, the ability to utilize various substation IEDs as synchrophasor measurement sources suggests that many high voltage substations will have this capability.

V. Control center level multi-area state estimator

It is unlikely that large areas of the grid will be equipped with redundant phasor measurements that guarantee observability any time soon [16]. On the other hand, many high voltage substations will gain this capability and certain sections of the higher voltage grid could be estimated with the above formulation. Thus integrating the classical and proposed state estimator together will be a very important step for transition. In this section, we describe a multi-area SE algorithm for a transitional SE in which portions of the formulation takes advantage of this advanced formulation while the rest of the grid is still solved in the traditional way. This multi-area state estimation method is similar with the previous algorithms introduced in [17]-[23] but with some novel ideas.

As introduced in part I Fig 7, we will partition the whole system into several linear and nonlinear areas. For each linear area, we can use the two-level linear state estimator to estimate subsystem states while for each nonlinear area, we can just use the traditional state estimator to do that. Suppose there are p linear areas and q nonlinear areas in the system, then we will have a new state estimation problem:

$$\begin{aligned}&\text{Min }\Big(\sum_{i=1}^{p}\tilde{\mathbf{r}}_i^T\tilde{\mathbf{W}}_i\tilde{\mathbf{r}}_i+\sum_{j=1}^{q}\mathbf{r}_j\mathbf{W}_j\mathbf{r}_j\Big)\\ &\text{s.t. }\tilde{\mathbf{z}}_i=\tilde{\mathbf{H}}_i\tilde{\mathbf{x}}_i+\tilde{\mathbf{r}}_i=\tilde{\mathbf{H}}_i[\tilde{\mathbf{x}}_{i,\text{int}}^T,\tilde{\mathbf{x}}_{i,\text{b}}^T]^T+\tilde{\mathbf{r}}_i\\ &\qquad\mathbf{z}_j=\mathbf{h}_j(\mathbf{x}_j)+\mathbf{r}_j=\mathbf{h}_j([\mathbf{x}_{j,\text{int}}^T,\mathbf{x}_{j,\text{b}}^T]^T)+\mathbf{r}_j\end{aligned} \tag{12}$$

where we divide the states in each area into internal states and boundary states.

Then the whole system state estimation problem will be divided into several subsystem state estimation problems. We can see from the flow chart shown in Fig 1, we divide the system and then use the linear state estimator to estimate the states in each linear area separately. For the ith linear area, we have the state estimation problem:

$$\begin{aligned}&\text{Min }\tilde{\mathbf{r}}_i^T\tilde{\mathbf{W}}_i\tilde{\mathbf{r}}_i\\ &\text{s.t. }\tilde{\mathbf{z}}_i=\tilde{\mathbf{H}}_i\tilde{\mathbf{x}}_i+\tilde{\mathbf{r}}_i=\tilde{\mathbf{H}}_i[\tilde{\mathbf{x}}_{i,\text{int}}^T,\tilde{\mathbf{x}}_{i,\text{b}}^T]^T+\tilde{\mathbf{r}}_i\end{aligned} \tag{13}$$

These linear areas only use phasor measurements as input, so can be run at higher periodicities, say 5 or 10 secs. Computation time is fast enough for this and the communication delays (discussed in Part I Appendix) do not introduce significant time-skews. However, synchronizing the measurement data is very important but the time stamping of the phasor data by GPS can enable this with an accuracy of a few micro-seconds.

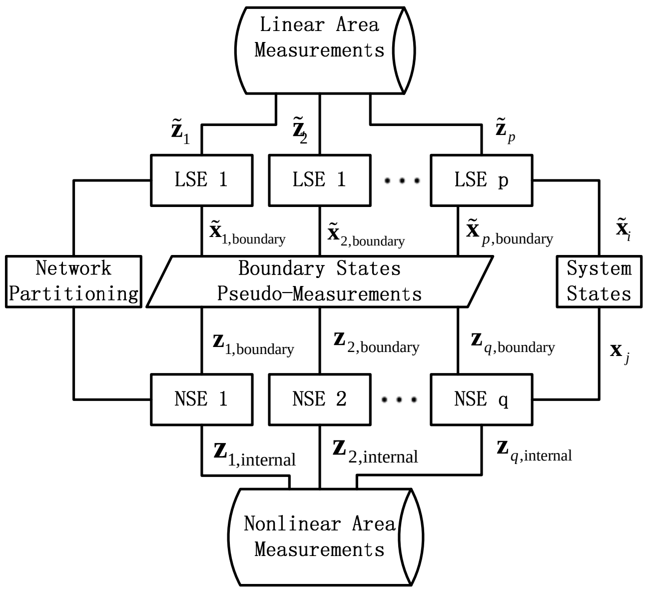

Fig 1. Flow chart of multi-area state estimation
(LSE-Linear State Estimator, NSE-Nonlinear State Estimator)

The multi-area state estimator, linear and non-linear areas combined, will be run at more traditional SE periodicities (in minutes) but synchronizing of the linear area solutions would be important here even though the nonlinear area measurements, which are not time stamped, cannot be as accurately synchronized. At the boundary buses we use the states estimated from the linear estimator as pseudo measurements with high accuracy for the nonlinear area estimate calculations. These pseudo measurements can be the estimated complex voltages and currents at the boundary buses as well as power and Var injections which can be calculated from the linear state estimates. For the jth nonlinear area, we have the state estimation problem:

$$\begin{aligned}&\text{Min } \mathbf{r}_j \mathbf{W}_j \mathbf{r}_j \\ &\text{s.t. } \mathbf{z}_j = \mathbf{h}_j(\mathbf{x}_j) + \mathbf{r}_j = \mathbf{h}_j([\mathbf{x}_{j,\text{int}}^T, \mathbf{x}_{j,\text{b}}^T]^T) + \mathbf{r}_j\end{aligned} \tag{14}$$

For the convenience of using in the traditional state estimator for each nonlinear area, we also generate the power flows on each boundary transmission line by the corresponding estimated currents and bus voltages from the linear state estimator. So the pseudo-measurements for the boundary perform as high accuracy PMU measurements set on the boundary bus. Besides, as we can provide each nonlinear state estimator phasor boundary bus voltages, we use the boundary bus as the reference bus for each nonlinear area. The reason we use this estimation order is that we can use the highly accurate and reliable linear SE to provide high-weight pseudo-measurements at the boundary buses of the nonlinear state estimators. Moreover, the linear part of this hybrid SE may be solved more frequently as phasor data is sampled at much higher rates than SCADA data for the traditional SE.

Although we used nonlinear state estimator in this case, the performance of this architecture will still be better than the traditional one because estimating the states of each small subsystem will be much faster and for each linear part, substation level linear state estimator will provide the reliable measurements.

VI. EXPERIMENTS

We use the IEEE-14 bus system as the test system (for a system diagram, see Part I Appendix, Fig. 8). In this system, the buses connected by transformers are considered to be in one substation. For each substation, we created the circuit breaker oriented single phase model.

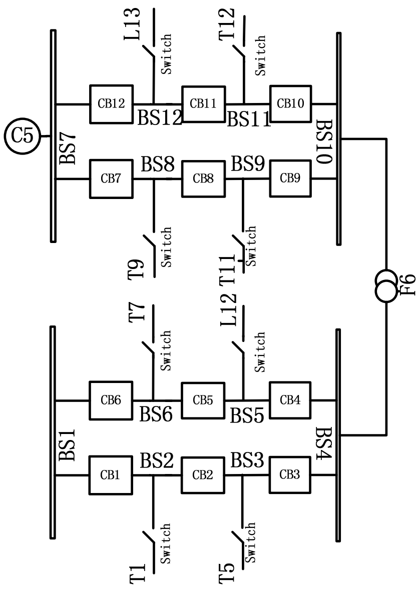

Fig 2. Circuit Breaker Oriented Model for Substation 5

For example, Fig. 2 shows the bus section-circuit breaker architecture of substation 5 of the IEEE 14 bus system. Fig 3 shows the substation 5 topology (if all circuit breakers are closed) that is calculated by the zero impedance current state estimator and sent to the control center.

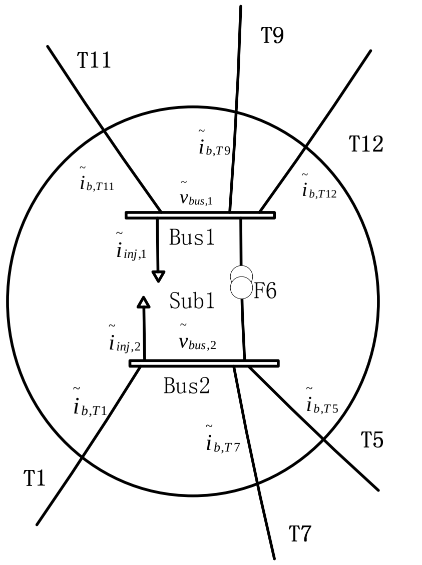

Fig 3. Topology for Substation 5 when all circuit breakers closed

We use a steady state power flow condition to generate the real-time measurement sets and consider these the true values. To make these measurements emulate real system measurements, Gaussian white noise is added to all measurements.

A. Substation Level State Estimation

For the kV-1 portion of substation in Fig 2, we generated a set of both the digital measurements (Table I) and analog measurements (Table II) (The generated data only works in this voltage level and does not fit the system level). The positive direction of circuit breaker current is counter-clockwise while that of bus injection current is injecting to the bus section.

TABLE I
DIGITAL MEASUREMENT SET FOR SUBSTATION LEVEL STATE ESTIMATOR

	True Value	Measurements
CB1 Status	Open	Open
CB2 Status	Closed	Closed
CB3 Status	Open	Closed (Bad Data)
CB4 Status	Closed	Closed
CB5 Status	Closed	Closed
CB6 Status	Open	Open

TABLE II
ANALOG MEASUREMENT SET FOR SUBSTATION LEVEL STATE ESTIMATOR

	True Value	Measurements
BS1 injection	0	0.0027 - j0.0174
BS2 injection	0.6 - j0.1	0.6081 - j0.0969
BS3 injection	-0.6 + j0.1	-0.5860 + j0.0805
BS4 injection	-0.4 + j0.1	-0.3919 + j0.0955
BS5 injection	-0.1 + j0.05	-0.1207 + j0.0318
BS6 injection	0.5 - j0.15	0.52569 - j0.1462
CB1 current	0	0.0118 - j0.0314
CB2 current	0.6 - j0.1	0.6129 - j0.0913
CB3 current	0	0.3841 - j0.1083 (Bad Data)

CB4 current	-0.4 + j0.1	-0.4009 + j0.0894
CB5 current	-0.5 + j0.15	-0.5054 + j0.1387
CB6 current	0	0.0086 - j0.0069

In addition to the Gaussian white noise with distribution is $N(0, 0.01^2)$, the measurements also include one status bad data and one analog bad data. Circuit breaker 1 & 3 & 6 are open with bad data of current on circuit breaker 3 and bad data of digital status on circuit breaker 3.

This mixture of digital and analog bad data is difficult to identify by the traditional state estimator. Although such a case is quite extreme and unlikely, it is a good test of the advantages of our algorithm. In contrast, we used the zero impedance state estimation algorithm to estimate the branch current and bus voltage at the same voltage level (kV-1). We can see from the normalized residuals shown in Table IV, the zero impedance current state estimator can detect and identify the bad data on the CB3 current measurement by the largest normalized residual analysis (here the largest normalized residue is 27.546 on the real part of CB3 measurement) and provide the control center with corrected inputs in Table III which is the estimation result with filter out the bad data. After we removed this measurement and re-estimated the states, the results and residuals become reasonable. We also see from Table III that the estimated current for CB3 is so close to zero that the status would be estimated to be open whereas the measurement status was closed. This situation as pointed out in section III.a is the unique case when the status cannot be determined with high probability, but fortunately this status does not affect the state estimator results.

TABLE III
ESTIMATED STATES OF THE ZERO IMPEDANCE CURRENT STATE ESTIMATOR

States	Analog	Digital
CB1	-0.0086 - j0.0194	Open
CB2	0.6094 - j0.1000	Closed
CB3	-0.0126 - j0.0117	Open
CB4	-0.3900 + j0.0917	Closed
CB5	-0.5107 + j0.1337	Closed
CB6	0.0098 - j0.0072	Open

TABLE IV
RESIDUAL ANALYSIS

	Residual		Normalized Residual	
	Real	Imaginary	Real	Imaginary
BS1 injection	-0.0053	-0.0026	0.7910	0.3794
BS2 injection	-0.0299	-0.0069	4.4562	1.0319
BS3 injection	-0.0914	0.0187	13.619	2.7891
BS4 injection	0.1129	-0.0344	16.835	5.133
BS5 injection	0.0371	-0.0199	5.5263	2.9604
BS6 injection	0.0145	-0.0077	2.162	1.1422
CB1 current	-0.0246	-0.0044	3.3153	0.5902
CB2 current	-0.0615	0.0256	8.2878	3.4563
CB3 current	0.2043	-0.0531	27.546	7.1659
CB4 current	-0.0759	0.0146	10.229	1.9653
CB5 current	-0.0226	0.0122	3.0431	1.6445
CB6 current	-0.0198	0.0051	2.6711	0.6902

There are analog and digital bad data together on circuit breaker 3. If we transferred these data to the control center, the traditional state estimator would have both a topology error as well as an analog bad data. From the results in Table III and IV, we can see that the substation level state estimator can detect and identify the analog errors

Numerous combinations of bad data were tested on this substation level linear state estimator. The following five sample test cases are presented:

Set 1: all the circuit breakers are closed without any bad data of analog measurements and the digital measurement of circuit breaker 1 is open, which is the bad status data.

Set 2: all the circuit breakers are closed with the bad data of currents on branch T1 and the bad data of status measurement on circuit breaker 1.

Set 3: circuit breaker 1 is open with the bad data of current on branch T5 and no bad data of digital measurement.

Set 4: circuit breaker 1 & 3 are open with bad data of current on T7 and the bad data of status measurement on circuit breaker 1.

Set 5: circuit breaker 1 & 3 & 6 are open with bad data of current on circuit breaker 3 and bad data of status measurement on circuit breaker 3.

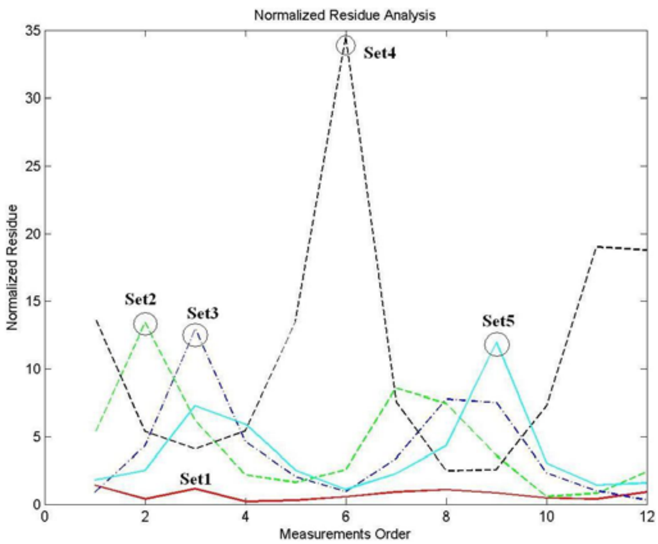

Fig 4. Normalized Residuals and Bad Data Identification

We show the normalized residuals estimated for each set in Fig 4. In Fig 4, horizontal axis represents the measurements in order, and the vertical axis represents the normalized residual of each measurement. For the convenient of analyzing, we use different types of lines to show different data sets. Then each circle on each line represents the largest normalized residue (the larger one of the real part and imaginary part), which corresponds to bad data in that data set. We can see that the substation level linear state estimator can decouple the analog and digital bad data, detect, and identify each of them.

Assume that the redundancy of the voltage sensors is enough, which means there will be at least one PMU for each bus, we can estimate each bus voltage by the weighted average value of the voltage measurements for each bus. After that we can provide the bus voltages, branch currents, injection currents and substation topology to the control center.

B. Comparison of Proposed and Traditional State Estimation

In this part, we will compare the proposed two-level linear state estimator and the traditional state estimator in the presence of bad data. We also compare these with a multi-area version in which the high voltage level area of the system (substation 1 to substation 5) has enough synchrophasor measurements to be solved by the linear equations while the low voltage level area (substation 4 to substation 10) is represented by nonlinear equations as in the traditional SE. The linear and non-linear areas are connected by two boundary buses 6 & 9, and as can be expected the performance of this multi-area state estimator falls in between the proposed linear SE and the traditional nonlinear SE.

1) Experiment 1: No Digital Bad Data

In the first set of experiments, we assume that all the statuses of the circuit breakers are closed, which means the system topology is just the same with the IEEE-14 bus power flow file. So we can compare the performance between the two-level linear state estimator and the traditional one when only analog bad data exist in the measurement set.

a) One analog bad data

Assume that there is just one bad data which is the branch measurement on transmission line T1. All the three estimators – two-level linear SE, traditional SE and multiple-area SE - can detect the bad data, identify it, and provide the correct result after deleting it from the measurements set. Table V shows the three SE solutions and even though the solutions are somewhat different because the measurement sets are different, all three can identify the single bad data correctly.

TABLE V
RESULTS COMPARISON WITH ONE ANALOG BAD DATA

	Voltage Magnitude (p.u.)			
	Linear SE	M-A SE	Traditional SE	True
Bus1	1.0597	1.0606	1.0594	1.06
Bus2	1.0447	1.0454	1.0472	1.045
Bus3	1.0100	1.0102	1.0100	1.01
Bus4	1.0186	1.0194	1.0222	1.019
Bus5	1.0196	1.0204	1.0218	1.02
Bus6	1.0694	1.0699	1.0783	1.07
Bus7	1.0613	1.0624	1.0663	1.062
Bus8	1.0892	1.0905	1.0874	1.09
Bus9	1.0552	1.0562	1.0522	1.056
Bus10	1.0502	1.0448	1.0523	1.051
Bus11	1.0562	1.0461	1.0635	1.057
Bus12	1.0544	1.0530	1.0558	1.055
Bus13	1.0494	1.0474	1.0565	1.05
Bus14	1.0354	1.0322	1.0403	1.036
	Voltage Angle (RAD)			
	Linear SE	M-A SE	Traditional SE	True
Bus1	0	0	0	0
Bus2	-0.0869	-0.0868	-0.0862	-0.0869
Bus3	-0.2220	-0.2217	-0.2206	-0.222
Bus4	-0.1803	-0.1800	-0.1785	-0.1803
Bus5	-0.1533	-0.1530	-0.1514	-0.1532
Bus6	-0.2486	-0.2480	-0.2412	-0.2482
Bus7	-0.2335	-0.2330	-0.2280	-0.2334
Bus8	-0.2333	-0.2327	-0.2263	-0.2332
Bus9	-0.2610	-0.2605	-0.2545	-0.2608
Bus10	-0.2638	-0.2676	-0.2558	-0.2635
Bus11	-0.2585	-0.2610	-0.2494	-0.2581
Bus12	-0.2635	-0.2669	-0.2543	-0.263
Bus13	-0.2651	-0.2662	-0.2550	-0.2646
Bus14	-0.2804	-0.2820	-0.2709	-0.28

Comments about the results: Theoretically, all three state estimators should be as accurate as each other. The accuracy difference is because accuracy for the phasor measurements is higher than that of the RTU measurements.

b) Multiple analog bad data

If there are multiple coupled bad data in the measurement set, the traditional state estimator often has difficulty in identifying the bad data. Many advanced methods for multiple analog bad data identification have been suggested in the literature [24]-[28], but their application has been limited and methods other than the usual hypothesis testing are not used in practice. The two-level linear state estimator can handle it very well because it can filter out the bad data at the substation level with the help of the substation measurement redundancy. This was already demonstrated in the experiments described about in section VI.A.

2) Experiment 2: Digital Bad Data included

In this part, we will introduce a bad data in the digital measurements set coupled with the analog bad data. As shown before the two-level linear state estimator can also identify the status bad data in the substation level easily while the traditional state estimator has even more difficulty with status errors. In the experiment, we use one bad analog data and one bad status data as in set 5 of section VI.A.

TABLE VI
RESULTS OF LINEAR STATE ESTIMATOR WITH ONE ANALOG BAD DATA AND ONE DIGITAL BAD DATA

	Voltage Magnitude (p.u.)		
	Linear SE	Traditional SE	True
Bus1	1.0679	1.1451	1.0682
Bus2	1.0448	1.0525	1.045
Bus3	1.0190	1.1382	1.0194
Bus4	1.0018	1.1474	1.0019
Bus5	0.9971	1.1153	0.9972
Bus6	1.0087	1.0086	1.0091
Bus7	1.0234	1.0947	1.0235
Bus8	1.0606	1.0968	1.0607
Bus9	1.0132	1.0774	1.0132
Bus10	1.0054	1.0623	1.0055
Bus11	1.0040	1.0331	1.0042
Bus12	0.9953	1.0124	0.9957
Bus13	0.9923	1.0173	0.9927
Bus14	0.9866	1.0275	0.9868
Bus15	1.0597		1.0599
	Voltage Angle (RAD)		
	Linear SE	Traditional SE	True
Bus1	0	0	0
Bus2	-0.0945	-0.0369	-0.0951
Bus3	-0.2738	-0.1726	-0.2744
Bus4	-0.2920	-0.1102	-0.2924
Bus5	-0.3080	-0.0776	-0.3084
Bus6	-0.3853	-0.2136	-0.3855
Bus7	-0.3504	-0.1516	-0.3507
Bus8	-0.3505	-0.1849	-0.3507

Bus9	-0.3806	-0.1821	-0.3810
Bus10	-0.3861	-0.1932	-0.3865
Bus11	-0.3878	-0.2111	-0.3881
Bus12	-0.3981	-0.2345	-0.3985
Bus13	-0.3985	-0.2345	-0.3988
Bus14	-0.4049	-0.2007	-0.4052
Bus15	-0.0531		-0.0538

For the traditional state estimator, analog measurements set is constituted by power flows, current and voltage magnitudes. We can see from Fig 5 that the weighted sum of squares of the residuals when solving the traditional SE appears to converge but the value is actually too high for this to be considered a converged solution. In Table VI, the solutions for the linear and traditional SE show that the traditional SE did not converge to anywhere near the true solution. This non-convergence is a common occurrence in traditional state estimators when status errors result in wrong topology for the system as in this case. Notice that the traditional SE did not detect the split bus and the extra bus 15 was not formed.

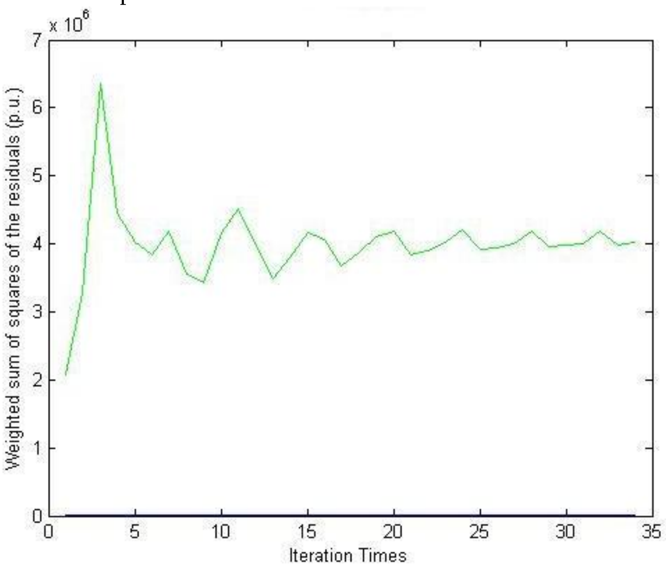

Fig 5. Weighted Sum of Squares of the Residuals per Iteration

These experiments show that the major advantage of such a two-level linear SE is its ability to detect, identify and discard bad data. The traditional SE solution is vulnerable to bad data, especially bad status data which can even prevent it from convergence.

VII. Conclusions

The new synchrophasor measurement units are expected to bring a revolution in power system applications. We propose in this paper, a decentralized two-level linear state estimator based on these phasor measurements. We first use the zero impedance state estimator to estimate the substation level analog states, digital states, and substation topologies. Then we transfer these filtered or estimated substation phasor measurement data with the topology of the substation to the control center instead of the raw data sent nowadays through the SCADA system. Thus we can use this phasor data and the substation topologies to build the system topology and estimate the power system states linearly.

The advantages of this two-level linear state estimator include:

(1) A linear solution to the system state that always guarantees a solution.

(2) Pre-filtering of the substation real-time data at the substation to provide more accurate input to the system level state estimation calculations. Thus, the bad data detection, identification and elimination is done before the state estimation calculation rather than after, guaranteeing the accuracy of the calculated system state.

(3) The substation level calculations are not only linear but also distributed over many substations, thus making this processing very fast. The topology processing and bad data detection at the substation level are also much simpler from a computational viewpoint.

(4) As the calculations are distributed between the substations and the control center, the database for the static data used in the calculations are also distributed.. Instead of one very large database at the control center, the substation information can be stored at each substation making the updating of these substation databases easier.

We also have described the transitional two level linear state estimator which can be used before all the substations are synchronized. We believe that as synchronized phasor measurements proliferate in the power grid, this state estimation system will eliminate many of the problems plaguing today's state estimators guaranteeing solutions that are more accurate. For further developments that to improve the state estimator, more accurate measurements are needed but it is not enough, in order to have a reliable SE, the consistency between the accuracy of measurements and the accuracy of models, must be ensured. A precise review of standard approximate models must be done.

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